Design and Analysis of Coordinated Access Schemes for Code-Limited Optical-CDMA Networks

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Abstract- This paper investigates code assignment and transmission scheduling schemes for code-limited optical code division multiple access (O-CDMA) networks, where the number of nodes exceeds the number of codes. We present an architectural design of a coordinated access scheme based on a centralized gated-polling controller, and evaluate its performance in terms of scheduling delays and achievable throughputs. In particular, this paper proposes a list-scheduling algorithm for code assignment and analyzes its near-optimal performance. With theoretic analyses and simulations, we derive the relationships among system capacity, traffic load, and network performance, and establish an analytical framework to address practical issues regarding O-CDMA network planning. Performance results indicate that the code-limited system can achieve high throughputs at the expense of scheduling delays. The capacity requirement of the control channel is also investigated to ensure system stability. These results provide an in-depth insight into network dynamics of code-limited systems and can serve as guidelines for network planning and algorithm design.

I. INTRODUCTION

Optical local access networks typically use wavelength-division multiplexing (WDM) and time-division multiplexing (TDM) technologies that require deterministic wavelength and time domain assignment. Recently, optical code-division multiple-access (O-CDMA) [1-5] has attracted much attention as a more flexible multiple access technology for optical networks. The O-CDMA technology promises many desirable features not available in WDM or TDM networks, including: format-independent physical layer security, low probability of detection or interception, anti-jamming capability against wavelength specific jammers, and potential covert overlay on existing optical networks. In O-CDMA networks, different user signals may overlap both in time and frequency domains in a common medium; multiple-access is achieved by assigning different, minimally interfering code sequences to different transceiver pairs. Instead of relying on fixed WDM or TDM based assignments, O-CDMA can utilize optical codes to achieve truly flexible access of large network capacity.

O-CDMA access networks operate in a broadcast-and-select manner, where all inputs from various nodes are combined in a passive star coupler and the mixed optical signal is broadcast to all users. Each node is equipped with a transceiver for data transmission and detection by encoding or decoding messages with the right code. We can view this type of network as a multiple parallel channel system, with each channel corresponding to an O-CDMA code linking a pair of transceivers. Here, the main design issue for packet-oriented O-CDMA networks is how to control access to these code channels, and the solutions to this problem form the set of medium access schemes. Previous studies [6-9] on O-CDMA mainly focus on random access schemes with fixed code assignment, where the number of users is typically limited by the number of available codes. Our previous study [6] has shown that the achievable throughput in such a system is relatively low because of channel collision or destination contention. Future advanced optical access networks are expected to support a large number of users, and there may not be sufficiently many codes available for fixed code assignment. When the number of users is larger than the number of codes, the network becomes a code-limited system so that it has to explore code sharing and coordinated access for contention-free transmission. In a code-limited network, the realization of coordinated access entails efficient schemes for code assignment and transmission scheduling, whose design and analysis are the focus of this paper.

This paper discusses architectural designs and performance analyses of code-limited O-CDMA networks with coordinated access schemes. We describe a coordinated access scheme based on a centralized gated-polling controller, and evaluate its delay-throughput performance. In particular, we propose a dynamic code assignment and scheduling algorithm, and demonstrate its limit-approaching performance. Through theoretical analyses and simulations, we determine the relationships among system capacity, traffic load and network performance, and lay out an analytical framework to address practical issues regarding O-CDMA network planning. The capacity requirement of the control channel is also analyzed to ensure system stability.

This paper is organized as follows. Section II describes the system architecture of O-CDMA networks with coordinated access schemes. Section III presents a list-scheduling algorithm for code assignment and analyzes its performance. Section IV conducts the delay-throughput analysis for code-limited systems. Section V investigates the capacity requirement of the control channel in a stable system. This paper concludes with a summary in Section VI.

II. ARCHITECTURE OF COORDINATED ACCESS SCHEMES

Fig. 1 (a) shows an O-CDMA network with coordinated access schemes. The network employs a centralized controller for code assignment and transmission scheduling, and uses a dedicated out-of-band control channel for pre-transmission control and coordination. Each node has a fixed control transceiver to exchange control messages with the

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controller. In addition, each node has one tunable transmitter and one tunable receiver (namely, TTTR architecture) to access data channels. Fig. 1(b) shows the nodal architecture of an end-node. Coordinated access control can be seen as a continuous and pipelined process involving three stages: 1) **Request**: the nodes send their access requests to the centralized controller; 2) **Scheduling**: the controller schedules these requests for contention-free transmission; and 3) **Notification**: the controller informs the nodes when and on which code channels to transmit. Upon receiving their assignments, nodes immediately tune to the assigned codes for data transmission.

Consider a packet-slotted O-CDMA network with $N$ nodes and $C$ codes ($N > C$); the node set is defined as $\{1,2,\ldots,N\}$ and the code set as $\{1,2,\ldots,C\}$. The requests are stored in $N \times (N-1)$ queues at the controller, one per source-destination pair. A scheduling algorithm is implemented at the controller on a slot-by-slot basis where one “slot” duration is equal to the transmission time required for each data packet.

This work is concerned with a coordinated access scheme based on a gated-polling controller, which involves cyclic operations with a variable cycle length. At the beginning of each cycle, the controller polls all request queues and develops a demand set for the current cycle, which consists of head-of-line requests in each nonempty queue. Then, the controller performs transmission scheduling by assigning codes and time-slots to these requests. When this batch of requests has been completed scheduled, the controller will start its next cycle.

In the analysis, we use a random binary matrix $D$ to specify the requests to be scheduled for each cycle, with $D(s,d) = 1$ if the source-destination pair $(s,d)$ has such a request and $D(s,d) = 0$ otherwise. Assume that the random variable $\{D(s,d)\}$ are independent and identically distributed (i.i.d) with probability $\Pr(D(s,d)) = p$ when $s \neq d$, and that $D(s,d) = 0$ when $s = d$. Previous studies [13-15] have used this demand matrix for performance analyses under different network scenarios. The goal of this work is to derive a lower bound to the average schedule length of a generic scheduling algorithm, where the length of a schedule (denoted as $T$) is the number of slots that it takes to perform code-assignment for requests specified in $D$. The estimation involves two constraints:

(C1) during a given slot, each node can be scheduled for at most one transmission and at most one reception;
(C2) during a given slot, the system can be scheduled for at most $C$ transmissions, i.e., each code channels can carry at most one packet.

Consider the first constraint (C1) on packet transmission. This implies that the length of a schedule for $D$ will be at least

$$T_i^* = \max_{1 \leq i \leq N} (U_i),$$

where $U_i$ is the number of requests with node $i$ as the source that needs to be scheduled in the current cycle, i.e., the column weight of $D$: $U_i = \sum_{j=1}^{N} d_{ij}$. According to the definition of $D$, random variables $U_i$, $1 \leq i \leq N$, are statistically independent and are binomially distributed with parameters $N-1$ and $p$ (hereafter denoted as $\text{Bin}(N-1, p)$), where $N-1$ is the number of trials and $p$ is the probability of success at each trial). As in [13], for a set of i.i.d. binomial distributed variables $\{X_k\}$, $1 \leq k \leq L$, with $\text{Bin}(M, \varepsilon)$, the expected value of the maximum of $X_k$ is defined as

$$f(M, \varepsilon, L) = E(\max_{1 \leq i \leq L} X_i) = M - \sum_{m=0}^{M-1} \left( \sum_{j=m}^{M} \binom{M}{j} \varepsilon^j (1-\varepsilon)^{M-j} \right)^L.$$  

Based on this definition, we have

$$T_i^* = f(N-1, p, N).$$

III. SCHEDULING ALGORITHMS AND PERFORMANCE ANALYSIS
A. Schedule length analysis with a gated-polling controller

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Note that the same analysis can be carried out for the dual case regarding the first constraint (C1) on packet reception.

The constraint (C2) is imposed by the limited number of codes. Since all \( nx(N-1) \) queues will uniformly share these codes, the number of packets using code \( j \), \( \forall j \leq C \) follows a binomial distribution \( \text{Bin}(N(N-1)/C,p) \). According to the constraint (C2), the length of a schedule for \( D \) would be at least \( T^* = \max_{1 \leq j \leq C} (V_j) \), i.e.,

\[
T^* \geq T^*_2 = f(N(N-1)/C,p,C).
\]

Combined (3) and (4), we obtain a lower bound \( T^* \) to \( T \) as

\[
T \geq T^* (N,C,p) = \max(f(N-1,p,N),f(N(N-1)/C,p,C)).
\]

(5)

B. List scheduling for code assignment

We propose a list-scheduling algorithm operating at the gated-polling controller for code assignment. In general, the list scheduling is a greedy algorithm which starts by serving an arbitrary set of requests that do not violate the transmission constraints. Once a code is available, another request, which could be scheduled without violating the transmission constraints, is chosen at random to serve. The list-scheduling in the \( i \)-th cycle will deal with a binary matrix \( D_i \) with \( D_i(s,d) = 1 \) if the source-destination pair \((s,d)\) has an eligible request and \( D_i(s,d) = 0 \) otherwise. At the beginning of each slot in the \( i \)-th cycle, the algorithm will randomly select a row in \( D_i \) as the starting point and performs a row-wise search for the first request encountered which has not been served yet. Here, the row-wise search examines the elements in a row from left to right, and after one row have been examined, the procedure moves to the next row and starts the row-wise search again. If a request \( D_i(s,d) \) has been found, a code will be randomly selected from the code pool and be assigned to this request. Due to the constraint of a single transmitter and a single receiver, each node can be scheduled for at most one transmission and one reception during a given slot. Thus, other requests with the source \( s \) or the destination \( d \) will not be eligible for code-assignment in the current slot. This process is continued until either all requests have been served or all \( C \) codes have been assigned. If there are still un-served requests after the \( m \)-th slot, the algorithm will proceed with the \((m+1)\)-th slot and start anew with the row-wise search. In the case that all requests in \( D_i \) have been served after the \( m \)-th slot, the controller obtains a schedule with length \( m \) to clear \( D_i \), and will start its \((i+1)\)-th cycle with a new demand matrix \( D_{i+1} \). Note that at the end of each slot, the controller will organize a “grant” message and send out the code-assignment information through the control channel. The aforementioned leftmost-first row-wise search implies that the rightmost entries of the demand matrix are scheduled later on the average. Fairness can be achieved by alternating the search direction between leftmost-first and rightmost-first per cycle.

Fig. 2 presents an example of the list-scheduling algorithm. Consider a system with \( N = 4 \), \( C = 3 \) and traffic matrix \( D \). For simplicity, we assume that the row-wise search on \( D \) always starts from the first row in the leftmost-first direction and that the code assignment in each slot follows the order of \( \{1,2,3\} \). In this example, the list-scheduling algorithm produces a schedule with length 3.

We conduct simulations to evaluate the average schedule length under the list-scheduling algorithm. Fig. 3 shows results in the cases of \( C=30 \) and \( N=60,180,300 \) and Table 1 compares the results with the lower bound values. We observe that the achieved schedule length with list-scheduling is very close to the lower bound \( T^*(N,C,p) \), which is defined in (5). As Table 1 shows, the simulation performance is within 5% of the lower bound. These results indicate that the list-scheduling algorithm produces a near-optimal performance in term of the average schedule length.

<table>
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<th>( P )</th>
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<th>0.6</th>
<th>0.8</th>
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<td>84.35</td>
<td>106.50</td>
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<td>58.25</td>
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<td>((T_{LS}-T^<em>)/T^</em>)</td>
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<td>1.8%</td>
<td>3.3%</td>
<td>3.4%</td>
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IV. DELAY-THROUGHPUT ANALYSIS

A. Analysis of scheduling delay

The gated-polling controller will operate in a system with random request arrivals. The request streams arrive to the controller randomly and are stored in \( nx(N-1) \) request queues, one per source-destination pair. In each schedule cycle, the controller performs code-assignment and transmission scheduling for the head-of-line requests in each nonempty queue. This section will analyze the delay performance in such systems. The key metric of interest for a given request is the scheduling delay, \( W \), which measures the time that elapses from the arrival of the request to the beginning of the request service cycle.
Consider a system with parameters $N$, $C$, $p$, where the controller continuously schedules request demand matrices $\{D_i, i \geq 1\}$. Recall that $D_i(s,d)$, $s \neq d$, follows a Bernoulli distribution with probability $p$. Let the request rate per source-destination pair be $\lambda$ and the length of the schedule for $D_i$ be $T_i$, with the list-scheduling algorithm, and assume that the random variables $\{T_i, i \geq 1\}$ are identically distributed. In this setting, the total arrival rate to the controller is $N(N-1)\lambda$, while the overall departure rate is $N(N-1)p/\bar{T}$, where $\bar{T}$ is the average length of the schedule, i.e., $\bar{T} = E(T_i)$. By flow conservation, the departure rate is equal to the arrival rate in a stable queuing system. Thus,

$$\lambda = p/\bar{T}. \quad (6)$$

The analysis is concerned with the queue for an arbitrary source-destination pair $(s,d)$. Assume that requests arrive to the queue $(s,d)$ according to a Poisson process with the rate $\lambda$. For the gated-polling controller, a schedule $i$ will complete its code-assignment for a request matrix $D_i$ with a schedule length $T_i$. From the perspective of the queue $(s,d)$, this transmission scheduling system can be viewed as an $M/G/1$ queue with vacations [13,16], where the controller serves a single request from the queue $(s,d)$ with service time $\{X_j, j \geq 1\}$ and takes a vacation of random duration of $\{Y_k, k \geq 1\}$. If there are requests in the queue $(s,d)$ at the beginning of a schedule $i$, i.e., $D_i(s,d) = 1$, the controller will take $T_i$ to serve the head-of-line request of queue $(s,d)$. On the other hand, when the queue $(s,d)$ has no request at the beginning of a schedule $i$, i.e., $D_i(s,d) = 0$, it will see that the controller offers no service and takes a vacation with the duration $T_i$. This vacation model takes into account of the fact that a new arrival to an empty queue cannot be served in the current schedule if it arrived after the moment the schedule has started. In this model, a new request arrival has to wait in queue for the completion of the current service or vacation and then for the completion of all service requests already in the queue. As in [16], the expected waiting time in an $M/G/1$ vacation queue is

$$\bar{W} = \frac{X^2}{2(1-\lambda X)} + \frac{Y^2}{2\bar{T}}, \quad (7)$$

where $X$ and $X^2$ are the mean and the second order moment of the service time while $Y$ and $Y^2$ are the mean and the second moments of the vacation time. Since $\{X_j, j \geq 1\}$ and $\{Y_k, k \geq 1\}$ are a subset of $\{T_i, i \geq 1\}$, both follow the same distribution as $\{T_i, i \geq 1\}$ in the limit case. Thus, we obtain

$$\bar{W} = \frac{T^2}{2\bar{T}(1-\lambda \bar{T})}, \quad (8)$$

where $T^2$ is the second moment of the schedule length. Using (6), we get

$$\bar{W} = \frac{T^2}{2\bar{T}(1-p)}. \quad (9)$$

We obtain the first two moments of $T$ from the simulations to evaluate (9). Fig. 4 shows sample results for the average scheduling delay as a function of $p$ for $N = 60$, 180, 300 and $C = 30$. For a given $p$, the system with a large number of nodes has a longer scheduling delay. For example, at $p = 0.5$, the scheduling delay is about 72 and 571 for $N = 60$ and 180, respectively, and it increases to about 1551 in the case of $N = 300$.

![Fig. 4. Average scheduling delay (C=30).](image)

B. Throughput-delay analysis

For a system with $N$, $C$, and $p$, we define the achievable throughput per code channel as a function of $p$:

$$TH = \frac{N(N-1)p}{T_C}. \quad (10)$$

By varying $p$, equations (9) and (10) jointly provide the delay-throughput relationship for a given $N$ and $C$. Numerically, Fig. 5 shows the delay-throughput relationship for the case of $N=300$ and $C=30$, indicating that the code-limited system is able to achieve high throughputs at the expense of additional scheduling delay. More notably, there is an abrupt increase in scheduling delay when the achievable throughput exceeds 0.9. Fig. 6 and Fig. 7 show the throughput and delay performance with different numbers of codes $C$ as $N$ is fixed to 300. The system with a smaller $C$ leads to a higher throughput per code channel, but incurs long delay to spread out transmissions in time. For instance, in the case of $p = 0.5$ and $N = 300$, the throughput and delay are about 0.928 and 537, respectively when $C = 90$. For $C = 30$, the throughput increases to be about 0.964, while delay becomes about 1551. These findings indicate that the delay-throughput performance heavily depend on the system parameters $N$, $C$, and $P$. Thus, in designing coordinated access schemes for O-CDMA networks, one key issue is how to select an appropriate combination of $N$ and $C$ under $p$ by balancing the tradeoff between the achievable throughputs and the incurred scheduling delay. In practice, this requires a detailed analysis on network dynamics, while taking into consideration the underlying application scenarios, performance requirements, and realistic traffic patterns, all of which are important subjects for further studies. The relationship expressed in (9) and (10) provides basic guidelines in carrying out such further studies.
Fig. 5. Throughput-delay performance for coordinated access schemes ($N=300$, $C=30$).

Fig. 6. Achievable throughput for coordinated access schemes with different number of codes ($N=300$).

Fig. 7. Average scheduling delay for coordinated access schemes with different number of codes ($N=300$).

V. CONTROL CHANNEL CAPACITY IN A STABLE SYSTEM

The coordinated access scheme relies on a control channel to exchange control messages between the controller and end-nodes. This section analyzes the capacity requirement of the control channel in a stable system.

Consider a simple TDM control channel where the time axis is divided into $N\times(N-1)$-slot frames with one slot dedicated to each source-destination pair, and let the duration of a slot $S_c$ be the transmission time of a control packet for transmission request. To examine the capacity requirement of the control channel, the code-limited system can be viewed as a two-stage tandem queueing system; the first stage is for the TDM control channel and the second for the list-scheduling-based code assignment. In a system with $N$, $C$, and $p$, the average output rate of this tandem system (i.e., the departure rate of the requests) is

$$r_{out} = \frac{N(N-1)p}{T(N, C, p)S_d},$$

where $T(N, C, p)$ is the average length of the schedule, and $S_d$ is the transmission time needed by each data packet. The constraint of the flow conservation in this tandem system implies that the average input rate $r_{in}$ equals $r_{out}$. Thus, the average request arrival rate for a given source-destination pair is:

$$\lambda = \frac{r_{in}}{N(N-1)} = \frac{p}{T(N, C, p)S_d}. \quad (11)$$

With a generic request arrival process with rate of $\lambda$, we see that the queue for a single source-destination pair in the TDM control channel is a $G/D/1$ queue with vacations [16], where the average service rate is $\mu = 1/(N(N-1)S_c)$. To maintain the stability of this $G/D/1$ queue, it is required that $\lambda < \mu$, i.e.,

$$S_c < g(N, C, p)S_d, \quad (12)$$

where $g(N, c, p) = T(N, C, p)/(N(N-1)p)$.

Fig. 8. $g(N, c, p)$: a monotonically decreasing function with $p$.

It is worth noting that $g(N, c, p)$ is monotonically decreasing with $p$. Fig. 8 illustrates an example with $N=180$ and $C=30$. For $p \in (0,1]$, the minimum value of $g(N, C, p)$ is achieved when $p=1$. In this case, $T(N, C, 1) = \lceil N(N-1)/C \rceil$ and we have a tight lower bound for $g(N, C, p)$: $g(N, C, p) \geq 1/C$. Thus, we obtain a tight and simple relationship between $S_c$ and $S_d$ as:

$$S_c < S_d / C. \quad (13)$$

This constraint is the guarantee that the TDM control channel will not become the bottleneck even when the network is fully loaded. Note that the maximum capacity of the system is $C \times T_{db}$, where $T_{db}$ is the capacity of a single data channel.
We present several numerical examples to illustrate the impacts and the implications of the above constraint. Assume that the control channel capacity is the same as a single data channel and that a control packet contains only two fields: a source address and a destination address. In a 1000-station system with 30 available codes (i.e., \( N = 1000 \) and \( C = 30 \)), the address field would be at least 10 bits in order to provide an address space which is large enough to accommodate 1000 stations, and thus the length of a control packet should be at least 20 bits. With \( C = 30 \), the constraint relationship in (13) indicates that the length of a data packet would be at least 600 bits. Another example considers the case when the control channel operates at a lower speed than that of a data channel. If the ratio of capacities between the data channel and the control channel is 10, the length of a data packet would be at least 6000 bits.

VI. CONCLUSION

In this work, we presented an investigation into architectural design and performance evaluation of code-limited O-CDMA networks. We discussed a coordinated access scheme based on the centralized gated-polling controller, and evaluated its delay-throughput performance. The proposed list-scheduling algorithm for code assignment achieves near-optimal performance in term of average schedule length. Performance results indicated that the code-limited system is able to achieve high throughputs at the expense of scheduling delays. Through theoretical analyses and simulations, we determined the relationships among system capacity, traffic load, and network performance, which establish a framework to address practical issues regarding O-CDMA network planning. The capacity relationship between the control channel and the data channel was also derived to ensure system stability. These analyses provide an in-depth insight into understanding network dynamics of code-limited systems and offer helpful guidelines for O-CDMA network planning and system design. Extension of these analyses to the connection-oriented networks is currently underway.

REFERENCE